

Finite Element Based Member Stiffness Evaluation of Axisymmetric Bolted Joint

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Abstract:

For a reliable design of bolted joints, it is necessary to evaluate the actual fraction of the external load transmitted through the bolt. The stiffness of the bolt and the member of the joint decide the fractions of external load shared by the bolt and the member. Bolt stiffness can be evaluated simply by assuming the load flow to be uniform across the thickness and the deformation is homogeneous. Then, bolt may be modeled as a tension member and the stiffness can be easily evaluated. But, the evaluation of the member stiffness is difficult because of the heterogeneous deformation. In the present work, joint materials are assumed to be isotropic and homogeneous, and linear elastic axisymmetric finite element analysis was performed to evaluate the member stiffness. Uniform displacement and uniform pressure assumptions are employed in idealizing the boundary conditions. Wide ranges of bolt sizes, joint thicknesses, and material properties are considered in the analysis to evaluate characteristic behavior of member stiffness. Empirical formulas for the member stiffness evaluation are proposed using dimensionless parameters. The results obtained are compared with the results available in the literature.

Keywords- bolted joints, member stiffness, finite element analysis, axisymmetric model.

INTRODUCTION:

In engineering and day to day life, many cases arise when we have to join the two members. There are many joining methods used in engineering, as welding, brazing, soldering, and bolting. Each method has its own characteristics and use. For a particular case one of method is good and reliable, but for another cases it may worst or impossible. Bolted joints are used in abundance within the mechanical design of machinery. Most of these joints are noncritical to the overall success of a machine's function; nevertheless, a certain number of these joints are extremely critical and require in depth analysis for determining their acceptability. In automobile field and machineries, the bolted joints are famous. In bolted joints screw threads are used for tightening of two members. For tightening of members in bolted joints it require a bolt, it may be hexagonal or square but mostly hexagonal is preferred, and nut. It is as shown in fig.1

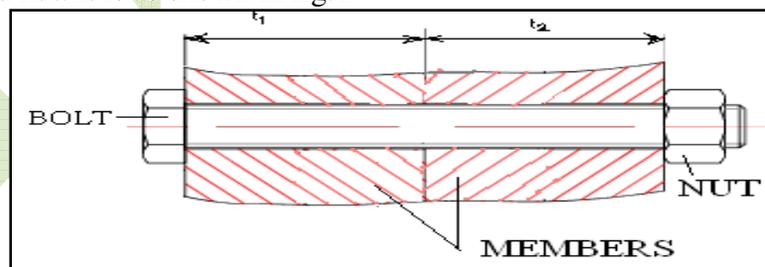


Fig.1 Bolted joint of a member

PRESTRESSING:

When it is necessary that the bolted member is joined enough strong to external force. So to tight the member prestressing is must. At the time of tightening extra torque is applied to the nut so that the member is compressed

and due to which compression force (F_m) is acting on the member and tensile force (F_b) is generated in the bolt. The main two advantages of prestressing are it increase the fatigue life of the joints and it create a locking effect to the joints, it means the joint will not lose easily. The resultant loads acting on bolt and member is

$$F_B = \frac{K_b}{K_b + K_m} P + F_i \tag{1}$$

$$F_m = \frac{K_m}{K_b + K_m} P - F_i \tag{2}$$

K_m and K_b are stiffness of the member and bolt material which is

$$K_m = \frac{F_m}{\delta m} = \frac{A_m E_m}{L} \tag{3}$$

$$K_b = \frac{F_b}{\delta b} = \frac{A_b E_b}{L} \tag{4}$$

STIFFNESS:

Stiffness is load per unit displacement of any elastic member. In the previous article we have discussed about the calculation of the stiffness of the member as

$$K_m = \frac{F_m}{\delta m} = \frac{A_b E_b}{L} \tag{5}$$

If there are more than one member in joined by nut-bolt it assumed to be spring structure in series as shown in fig.2.

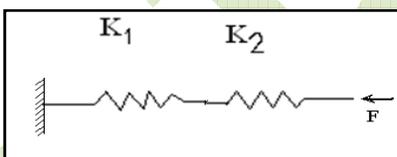


Fig.2 Stiffness model of member

The overall stiffness can be calculated,

$$\frac{1}{K} = \frac{1}{K_1} + \frac{1}{K_2} \tag{6}$$

This formula is used when there is use of gasket in between the members and gasket is having stiffness less than other members, then other can be neglected for all practical purposes and only gasket stiffness is used. But if there is no gasket and if only a two member are joined by nut-bolt then it is difficult to calculate the joint stiffness, except by actual experimentation. Because deflection and force, area acting on the member is non uniform, it may be more at near the head and nut and low at the interface of two members.

In such cases where area under the compression are not known Prof. Charles Michke suggest the concept of pressure cone. Many analytical methods were proposed to calculate the area under the calculation but the pressure cone method is dominated. To model the actual system the conical shaped system is proposed. Prof. Michke propose the cone angle of 45°

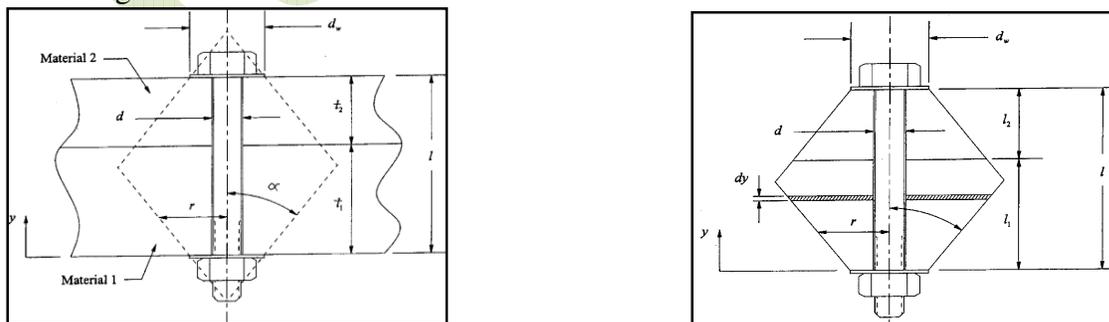


Fig.3 Cone geometry of bolted member

Fig.3 shows cone geometry, it has washer diameter d_w , on both sides and joint of equal thickness member, this is assumed to simplify the case. But in actual there may be member of unequal thickness and may be more than one member of different modulus of elasticity. Shiglay [1] in his book assume $\alpha=45^\circ$. Later the same author proposed the angle as $\alpha=30^\circ$. Verein Deutscher Ingenieur procedure consider it as $\alpha=30^\circ$. The stiffness of frustum is given by Shiglay as

$$K_m = \frac{\pi E m d \tan \alpha}{2 \ln \left\{ \frac{(d_w + L \tan \alpha - d)(d_w + d)}{(d_w + L \tan \alpha + d)(d_w - d)} \right\}} \quad (7)$$

Assuming washer diameter $d_w=1.5d$, & $\alpha=30^\circ$ as best value

$$K_m = \frac{0.577 \pi E m d \tan \alpha}{2 \ln \left\{ 5 X^{\frac{(0.577 L + 0.5d)}{(0.577 L + 2.5d)}} \right\}} \quad (8)$$

The distinguishing characteristics of this system are cone angle ' α ', washer diameter ' d_w ', bolt diameter ' d ' and clamped material thickness ' L '. Perhaps the most unsettled topic concerning this model has been the selection for the cone angle ' α ', as the different author use different angle, no one has discussed relative important of choosing the proper value for this parameter. Verein Deutscher Ingenieur (VDI) procedure assumes α to be 30° . Juvinall and Marshek proposed the following expression to estimate the effective clamping area, A_m , of the member.

$$A_m = \frac{\pi}{4} \left[\left(\frac{d_2 - d_1}{2} \right)^2 - d_h \right] \quad (9)$$

Where $d_h=d$ for small clearance, $d_1=1.5d$, $d_2= d_1+L \tan \alpha$, and $\alpha=30^\circ$. Above equation is

$$A_m = d^2 + 0.68 d L + 0.065 L^2 \quad (10)$$

The stiffness of the member can be calculated by substituting above eq. in the equation of stiffness.

Apart from analytical models, many researchers used numerical methods such as finite element method to estimate member stiffness. Wileman et al performed axisymmetric finite element analysis (FEA) and proposed an exponential expression for member stiffness. In the analysis, the washer diameter d_w was assumed to be 1.5 times the diameter of the bolt. They considered the displacement of the uppermost node located on the center line of the washer for stiffness calculation. Finite element analyses for various aspect ratios (d_h/L) were carried out and finally an exponential relation for member stiffness evaluation was proposed. This is as below,

$$K_m = E m d h A e^{B d h / l} \quad (11)$$

Where,

A, B = material constants.

$A = 0.78952$ and $B = 0.62914$

Which valid for engineering materials such as steel, copper, aluminum, and cast iron. This formula is mostly applicable for the cases with $d_h/L \leq 2$ and shown to be quite simple and efficient method to calculate the member stiffness. Later many researchers used and developed the same technique to evaluate the member stiffness. The work is proposed an exponential relationship using no dimensional geometric and material parameters for member stiffness evaluation. The analysis was done using finite elements considering various bolt sizes, geometries, and different member materials. Most of the methods presented in the literature review have differences in the results. The differences in the results are mainly due to the assumptions made during the model development. This is value of pressure cone angle and confined uniform stress field in a particular region. Manning [3] showed the member stiffness depends on the choice of cone angle. The differences caused by various assumptions need higher safety factors for reliable design. Wileman formula, given in Eq. (11), is an attempt to handle the inaccuracies but the formula itself is restricted to limited variation of parameters. In this, we are doing an attempt to carry out systematic study and evaluation of member stiffness for various geometric and

material properties of the joint members. An ideal condition are considered where effects of member diameter, thread friction, contact friction between bolt head/washer and members, interface slip between members, and external shear loads are not considered in the present analysis; hence all such effects if present should be compensated by proper safety factor. The proposed formulas should not be applied for joints in which members may subject to very high external loads and tend to separate from each other.

METHOD OF APPROACH:

Our main aim is to propose a formula for member stiffness evaluation based on the results of linear finite element analysis. The member stiffness is calculated using finite element method and it is divided by a stiffness measure K_o to get a dimensionless quantity R , called “correction factor,” as given by

$$R = \frac{K(FEM)}{K_o} \tag{11}$$

$$K_o = \frac{\pi E_m (d_w^2 - d_h^2)}{4L} \tag{12}$$

Where,

K_o =the stiffness of hollow cylinder with internal and external diameters the same as that of the washer and length equal to the joint thickness.

If, $K_o = K_{FEM}$, then $R=1$

So we are comparing the K_{FEM} to the stiffness of the hollow cylinder with internal and external diameters the same as that of the washer. Correction factors for various joint geometries and material constants are obtained by detailed parametric finite element analysis and empirical relations to calculate the correction factor are proposed by least squares curve fitting.

Finite Element Modeling and Analysis:

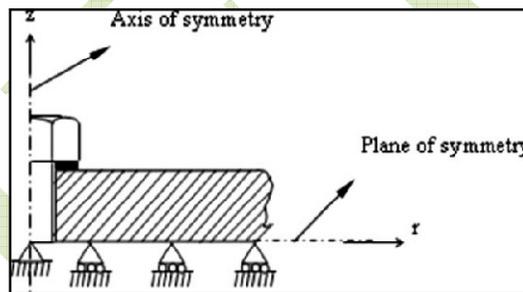
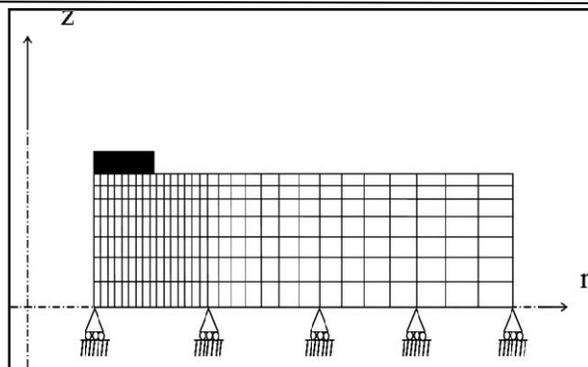


Fig. 4 Axisymmetric half model of the bolted joint

For the finite element modeling of the bolted joint, the assumptions made are

- 1) The geometry of the joint is assumed to be perfectly Axisymmetric.
- 2) The two members of the joint are assumed to be made of the same material and have the same thickness,
- 3) Also assumed to be in perfect contact without slippage.
- 4) The effect of screw threads is ignored.
- 5) The member material is assumed to be homogeneous, isotropic, and linearly elastic.

Due to the second assumption, the two members of the joint can be treated as a single continuous member and planar symmetry about the joint midplane is introduced in the model. Both the symmetries are incorporated in the model by applying proper symmetry constraints, as shown in Fig. 4. Commercial finite element software ANSYS is used for modeling and analysis. To avoid the edge effects, the outer diameter of the member is taken to be five times the diameter of the bolt hole. The model is meshed by eight noded axisymmetric quadrilateral elements. A schematic finite element discretization is shown in Fig.5. The member material is assumed to be homogeneous, isotropic, and linearly elastic with Young’s modulus, E_m , equal to 210 GPa, for all the analyses.



Bolt size	Bolt diameter d (mm)	Hole diameter $d_h, d_h = d + 2c$ (mm)	Washer diameter $d_w, d_w = 1.5d$ (mm)
M6	6	6.5	9
M8	8	8.5	12
M10	10	10.5	15
M12	12	13	18
M16	16	17	24
M20	20	21	30
M24	24	25	36
M30	30	31	45
M36	36	37	54

Fig. 5 Schematic finite element mesh

The outer diameter of the washer is taken to be 1.5 times the Diameter of the bolt, i.e., same as the diameter of the bolt head (distance between flats of metric hexagonal bolt head). Dimensions of various bolts and corresponding washers used in the analysis are given in Table 1.

BOUNDARY CONDITIONS:

- 1) $E_1 = E_2$
- 2) $t_1 = t_2 = l/2$
- 3) Plane of symmetry is steady i.e. plane of symmetry is remains plane after deformation.
- 4) Force applied at top

In this work, the load transfer is simulated by two different assumptions, which are explained below and the results obtained from both the assumptions are compared.

UDA:

This case corresponds to a joint in which the washer is highly rigid (hard washer) compared with the member. Due to this, the displacement of member under the washer is almost uniform. This condition is realized in the finite element analysis by applying a predetermined axially downward uniform displacement to the topmost nodes, which fall within the outer diameter of the washer. The sum of the upward reactions, F_r , of the nodes below the washer is divided by twice the applied displacement (due to symmetry) to get the stiffness of the member.

The stiffness is given by

$$K_{FEM} = \frac{F_r}{2\delta} \quad (13)$$

UPA:

This case corresponds to a joint in which the washer is soft compared with the member material. So in this case the contact pressure acting on the member surface is nearly uniform. In order to realize this condition, a predetermined uniform pressure is applied on the topmost nodes, which fall just within the washer outer diameter. Then the average member displacement is calculated by averaging the individual displacements of these nodes. The member stiffness is given by

$$K_{FEM} = \frac{pA_w}{2\delta_a} \quad (14)$$

Where A_w is the area under the washer and is given by

$$A_w = \frac{\pi}{4}(d_w^2 - d_h^2) \quad (15)$$

The average nodal displacement is used for calculation of K_{FEM} only for UPA because the displacement field under the washer contact area is varying in the radial direction with a maximum value at the bolt hole and a

minimum value at the outer edge of the washer. Longitudinal stiffness also varies along the radial direction. From the mean value theorem, for constant pressure boundary conditions, the average displacement gives the member stiffness in an average sense. Earlier, Lehnhoff et al. also used average displacement of nodes below the bolt head to calculate the member stiffness. The uniform displacement assumption (UDA) corresponds to the finite element boundary condition where the washer is assumed to be highly rigid compared with the member and the uniform pressure assumption (UPA) corresponds to the boundary condition where the washer is assumed to be very soft compared with the member. Plots of displacement contours of a 40 mm thick joint with M20 bolt with UDA and UPA are shown in Figs. 6 and 7, respectively.

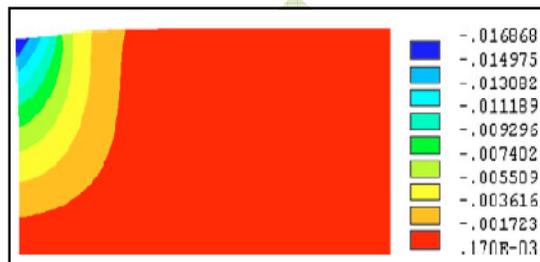
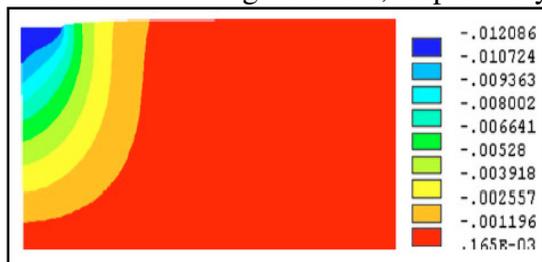


Fig.6 Displacement contours with UDA (M20 joint, L=40 m) Fig.7 Displacement contours with UPA (M20 joint, L=40 m)

In the actual joint neither the displacement nor the contact pressure below the washer is uniform. The idea behind using the assumptions for preload simulation is to calculate two

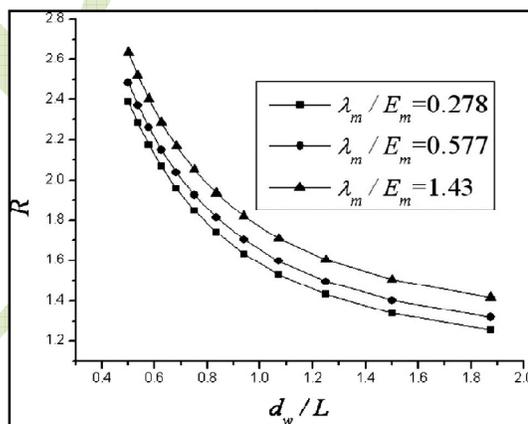
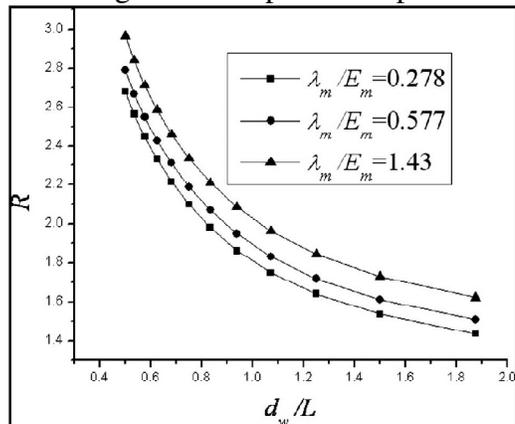


Fig. 8 Variation in R with dw/L for bolt size M20 (UDA) Fig. 9 Variation in R with dw/L for bolt size M20 (UPA)

different estimates of member stiffness, which will work as upper and lower bounds and the actual value of member stiffness will be in between the limits when the analysis is carried out with the actual washer stiffness. The assumptions of UDA and UPA may give (not proved in the present study) the upper and lower bounds of the member stiffness pertaining to the material rigidity of the washer relative to the member. In practice, the washer will have stiffness, which may not be very rigid or soft, and the rigidity will be of finite values. Convergence study is carried out on the initial finite element model by decreasing the element size and converged model with element edge length of 0.33 mm is used for the rest of the

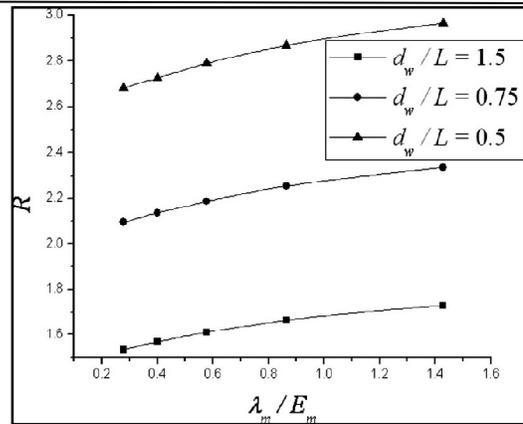
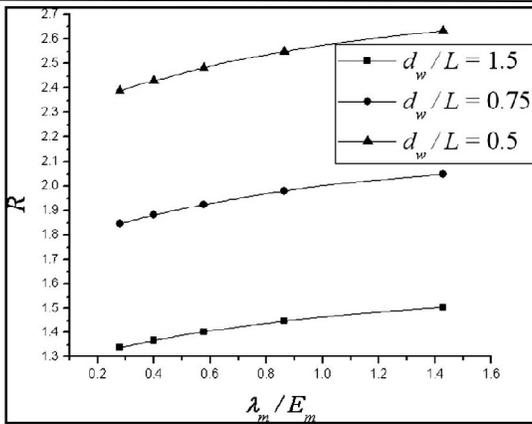


Fig.10 Variation of R with λ_m/E_m for bolt size M 20 (UDA) Fig.11 Variation in R with λ_m/E_m for bolt size M 20 (UPA) Analysis. The number of elements used in the analysis is not constant due to change in geometry. The number of elements used in the converged analyses ranges from 1000 to 40,859.

RESULTS AND DISCUSSION:

Parametric study is carried for nine different bolts ranging from M6 to M36 with joint thickness ranging from 16 mm to 60 mm with an increment of 4 mm and with five different Poisson ratio values varying from 0.2 to 0.4 with an increment of 0.05 for both UDA and UPA boundary conditions. The number of converged case studies conducted for each of UDA and UPA is 540. Correction factors are calculated for all the case studies using Eq.(11) used above. The finite element stiffness values for various bolt sizes and joint geometries are given in reference [4], but one of it are given in Tables 3.

Empirical Relation for Correction Factor: It is observed from the results of FEA that the stiffness of the member is independent on the dimensions of the bolt hole, washer diameter, and grip length of the joint. Along with these three factors, the Member stiffness is also dependent on Lamé’s constant of the member material, λ_m . This is because when the member is axially compressed by the load, the member below the washer will try to expand in radial direction due to Poisson’s effect and the material around the washer will restrict this expansion due to which radial stresses are induced in the member. These stresses, which influence the member stiffness, are observed to be proportional to Lamé’s constant. Therefore, the following dimensionless parametric group is used to express the correction factor.

$$R = f\left(\frac{d_w}{d_h}, \frac{d_w}{L}, \frac{\lambda_m}{E_m}\right) \tag{16}$$

As metric standard bolt dimensions are used throughout the analysis, d_w/d_h is nearly constant and retained in the expression only to represent the effects of bolt hole clearance. Variation in correction factor with d_w/L is shown in Figs. 6 and 7 for UDA and UPA, respectively. In each plot, it can be observed that the curves resemble exponential decay pattern. Figures 8 and 9 show that the correction factor increases with increase in λ_m/E_m (ratio is a function of Poisson’s ratio). Finally, an empirical relation for correction factor was proposed after observing the variation with individual variables of Eq. (16) and using a least squares curve fitting algorithm. During the initial curve fitting process, the curve was assumed to be of the form, exponential proposed by Wileman et al. In our modeling, three non dimensional parameters (d_w/d_h , d_w/L , and λ_m/E_m) are introduced to account for the wide range of geometric and material parameter variations. Later, the correction factor (R) variation is characterized for different geometrical non dimensional parameter variations for a given material property value. It was found that use of powers of non dimensional variables $(d_w/d_h)^{C1}(d_w/L)^{C2}$ fitted the curve well. The process was repeated for different material property values (λ_m/E_m). Finally, it was observed that the introduction of inverted hyperbolic function allowed fitting all the results into a single curve with minimal error (less than 3%). The final expression for correction factor is given by

$$R = C5 + C6 \exp(S) \tag{17}$$

Where S is an intermediate variable used for convenience of expression and it is given by

$$S = C_4 \sinh^{-1}((dw/dh)^{c1}(dw/L)^{c2}) + (\lambda M/EM)^{c3} \quad (18)$$

The constants used in Eqs(17) and (18) for UDA and UPA are given in Table 2. The maximum error in curve fitting is below 3% for both UDA and UPA. The member stiffness for any arbitrary joint geometry can now be calculated using the proposed empirical formula. However, it is very important to carefully review the inherent assumptions used to obtain this formula before using it, particularly the preload simulation methods using UDA and UPA. As explained earlier, both the assumptions are extreme idealizations of the actual phenomenon. Accordingly, the stiffness obtained by UDA always predicts the higher value for the member stiffness than that obtained by UPA. The selection of the appropriate assumption should be made depending on the problem at hand. The diameter of the washer is taken to be equal to the bolt head diameter (1.5 times the diameter of bolt) in the present work. As the diameter of the washer influences the member stiffness, these formulas should not be used if the difference between the washer diameter and the bolt head diameter is considerable. In the case of a joint without washer, formula can be used by replacing washer diameter by bolt head diameter. It is observed in the present work that the member stiffness is increased by 8–13% approximately with the increase in (λ_m/E_m) (as (λ_m/E_m) is a function of Poisson ratio) alone) from 0.28 ($\mu = 0.2$) to 1.43 ($\mu = 0.4$), see Figs. 10 and 11. This variation was accounted for in the proposed empirical formulas. In this work, the outer diameter of the member, d_m , is

taken to be five times the diameter of bolt hole, d_h , to avoid the edge effects. Except for very small values of d_w/L , increase in member diameter beyond 3.5 times d_h results in a negligible increase in stiffness, see Fig. 10. It is also evident from Fig. 10 that the member stiffness decreases steeply with a decrease in the member diameter beyond three times d_h . Hence the proposed formula can be safely used for member diameters starting from 3.5 times d_h .

Constant	UDA	UPA
C_1	-1.9690	-2.0417
C_2	-1.0831	-1.1605
C_3	0.051039	0.048737
C_4	0.69997	0.65097
C_5	-0.66075	-0.67007
C_6	0.69004	0.64828

CONCLUSIONS:

In the present work, the member stiffness of bolted joints is calculated using the axisymmetric finite element analysis and an empirical formula for the computation of member stiffness is proposed. The external load on the joint member is applied using UDA and UPA. Parametric study has been carried out for different bolts ranging from M6 to M36, joint thickness ranging from 16 mm to 60 mm, and Poisson ratio varying from 0.2 to 0.4. It is observed that the member stiffness varies by 8–13% with an increase in Poisson's ratio from 0.2 to 0.4. The proposed empirical formulas fit very well with the finite element simulations and the maximum deviation observed was less than 3%. Finally, the results are compared with the results obtained from previous methods in the literature and it is observed that the reported results overestimate the stiffness. The overestimation of the member stiffness for the conical clamp analytical model is mainly due to the cone angle selection. Selecting a lower cone angle may give better estimates of the stiffness.

NOMENCLATURE:

- A, B = material constants
- C = radial clearance
- D = nominal diameter of the bolt
- L = grip length/thickness of the joint
- P_{ext} = external load
- p = uniform pressure applied on the nodes under washer
- R = correction factor

- S = intermediate variable
 r, z = radial and axial coordinates
 α = semivertex angle of the frustum of pressure cone
 δ = boundary displacement applied on the nodes under washer
 μ = Poisson ratio
 A_b = nominal cross sectional area of the bolt shank
 A_m = effective clamping area
 A_w = area under the washer/bolt head in the case of no washer
 C_i = constants used in correction factor expression, the subscript i vary from 1 to 6
 d_h = diameter of bolt hole or washer inner diameter
 $(d_h = d + 2c)$
 d_m = outer diameter of the member
 d_w = washer outer diameter/bolt head diameter in the case of no washer
 d_1, d_2 = minor and major diameters of the conical frusta
 E_b = elastic modulus of bolt material
 E_m = elastic modulus of member material
 F_b = fraction of external load acting on bolt
 F_i = preload applied on the joint
 F_m = fraction of external load acting on members
 F_r = sum of reaction forces of nodes under washer in UDA
 K_b = bolt stiffness
 K_{FEM} = member stiffness, calculated by finite element analysis
 K_m = member stiffness
 K_o = stiffness of hollow cylinder with inner and outer diameters as that of washer and length equal to the thickness of the joint.
 δ_a = average displacement of nodes under washer
 δ_b = effective elongation of bolt due to preload
 δ_m = effective compression of member due to preload

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