

## TO STUDY THE EFFECT OF CRACK ON NATURAL FREQUENCY IN A CANTILEVER STRUCTURE BY USING EULER'S BEAM THEORY.

Mr Ganesh G. Gade

Department of Mechanical Engineering, Savitribai Phule University of Pune / PREC, Loni,  
Ahemadnagar, India

Mr.Amol L. Khatode

Department of Mechanical Engineering, Savitribai Phule University of Pune / SGOI, COE, Belhe,  
Pune, India

Miss Rajashree B. More

Department of Mechanical Engineering, Savitribai Phule University of Pune / SVCET, Rajuri,  
Pune, India

### ABSTRACT

The presence of cracks causes changes in the physical properties of a structure which introduces flexibility, and thus reducing the stiffness of the structure with an inherent reduction in modal natural frequencies. Consequently it leads to the change in the dynamic response of the beam. In this paper, A model for free vibration analysis of a beam with an open edge crack has been presented. Variations of natural frequencies due to crack at various locations and with varying crack depths have been studied. A parametric study has been carried out. The cracked beams with different boundary conditions have been analyzed. The results obtained by Euler's beam theory.

**KEYWORDS** – Beam; Free vibration; Crack; Natural frequencies;

### INTRODUCTION

Most of the members of engineering structures operate under loading conditions, which may cause damages or cracks in overstressed zones. The presence of cracks in a structural member, such as a beam, causes local variations in stiffness, the magnitude of which mainly depends on the location and depth of the cracks. The presence of cracks causes changes in the physical properties of a structure which in turn alter its dynamic response characteristics. The monitoring of the changes in the response parameters of a structure has been widely used for the assessment of structural integrity, performance and safety. Irregular variations in the measured vibration response characteristics have been observed depending upon whether the crack is closed, open or breathing during vibration.

The vibration behavior of cracked structures has been investigated by many researchers. The majority of published studies assume that the crack in a structural member always remains open during vibration. However, this assumption may not be valid when dynamic loadings are dominant. In such case, the crack breathes (opens and closes) regularly during vibration, inducing variations in the structural stiffness. These variations cause the structure to exhibit non-linear dynamic behavior. A beam with a breathing crack shows natural frequencies between those of a non-cracked beam and those of a faulty beam with an open crack.

In this paper, the natural frequencies of cracked and uncracked beams have been calculated using Euler's beam theory. Parametric study has been carried out on beams with crack at various crack depths and crack locations.

### LITERATURE REVIEW

Christides and Barr [1] developed a one-dimensional cracked beam theory at same level of approximation as Bernoulli-Euler beam theory. Ostachowicz and Krawczuk [2] presented a method of analysis of the effect of two open cracks upon the frequencies of the natural flexural vibrations in a cantilever beam. They replaced the crack section with a spring and then carried out modal analysis for each part of the beam using appropriate matching conditions at the location of the spring. Liang, Choy and Jialou Hu [3] presented an improved method of utilizing the weightless torsional spring model to determine the crack

location and magnitude in a beam structure. Dimarogonas [4] presented a review on the topic of vibration of cracked structures. His review contains vibration of cracked rotors, bars, beams, plates, pipes, blades and shells. Shen and Chu [5] and Chati, Rand and Mukherjee [6] extended the cracked beam theory to account for opening and closing of the crack, the so called "breathing crack" model. Kisa and Brandon [7] used a bilinear stiffness model for taking into account the stiffness changes of a cracked beam in the crack location. They have introduced a contact stiffness matrix in their finite element model for the simulation of the effect of the crack closure which was added to the initial stiffness matrix at the crack location in a half period of the beam vibration. Saavedra and Cuitino [8] and Chondros, Dimarogonas and Yao [9] evaluated the additional flexibility that the crack generates in its vicinity using fracture mechanics theory. Zheng et al [10] the natural frequencies and mode shapes of a cracked beam are obtained using the finite element method. An overall additional flexibility matrix, instead of the local additional flexibility matrix, is added to the flexibility matrix of the corresponding intact beam element to obtain the total flexibility matrix, and therefore the stiffness matrix. Zsolt huszar [11] presented the quasi periodic opening and closings of cracks were analyzed for vibrating reinforced concrete beams by laboratory experiments and by numeric simulation. The linear analysis supplied lower and upper bounds for the natural frequencies. Owolabi, Swamidias and Seshadri [12] carried out experiments to detect the presence of crack in beams, and determine its location and size. Yoon, In-Soo Son and Sung-Jin

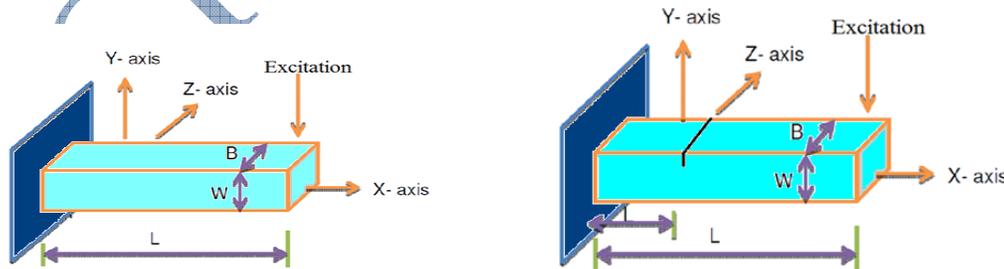
**MATERIAL & METHODOLOGY**

Structural steel beams have been considered for making specimens. The specimens were cut to size from readymade square bars. Total 05 specimens were cut to the size as length 700 mm and cross section area is 10mmX10mm. The modulus of elasticity and densities of beams have been measured to be 210GPa and 7850 Kg/m<sup>3</sup> respectively. Theoretical analysis is done by Euler's beam theory.

**GOVERNING EQUATION FOR FREE VIBRATION OF BEAM**

The cantilever beam with a transverse edge crack is clamped at left end, free at right end and has same cross section and same length like model in Fig.1 and 2. The Euler- Bernoulli beam model is assumed for the Theoretical formulation. The crack in this particular case is assumed to be an open surface crack and the damping is not being considered in this theory. Both single and double edged crack are considered for the formulation. The free bending vibration of a beam of a constant rectangular cross section having length l, width b, and depth w is given by the Euler's beam theory as follows: If the cross sectional dimensions of beam are small compared to its length, the system is known as Euler-Bernoulli beam. Only thin beams are treated in it. The differential equation for transverse vibration of thin uniform beam is obtained with the help of strength of materials. The beam has cross section area A, flexural rigidity EI and density of material  $\rho$ .

Consider the small element dx of beam is subjected to shear force Q and bending moment M, as shown in figure 3. While deriving mathematical expression for transverse vibration, it is assumed that there are no axial forces acting on the beam and effect of shear deflection is neglected. The deformation of beam is assumed due to moment and shear force.



**Fig. 1 Uncracked cantilever beam model      Fig. 2 Cracked cantilever beam model**

The net force acting on the element,

$$Q - \left( Q + \frac{\partial Q}{\partial x} dx \right) = dm * acceleration$$

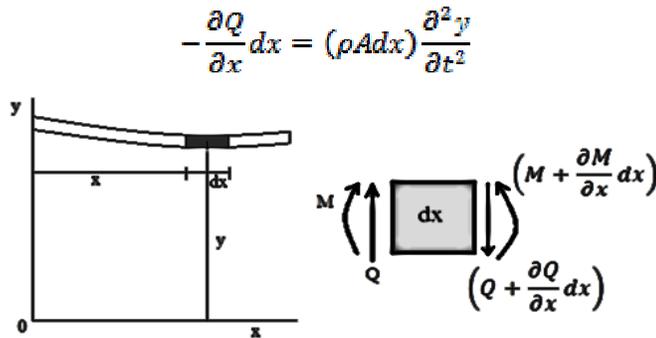


Fig. 3 Shear Force and Bending Moment acting on beam element

$$\frac{\partial Q}{\partial x} + \rho A \frac{\partial^2 y}{\partial t^2} = 0 \quad (\text{Equation 1})$$

Considering the moments about A, we get

$$M - \left( M + \frac{\partial M}{\partial x} dx \right) + \left( Q + \frac{\partial Q}{\partial x} dx \right) dx = 0$$

$$-\frac{\partial M}{\partial x} + Q + \frac{\partial Q}{\partial x} dx = 0$$

So  $Q - \frac{\partial M}{\partial x}$  higher order derivatives are neglected here  $\left( \frac{\partial Q}{\partial x} dx = 0 \right)$

$$\text{or } \frac{\partial Q}{\partial x} = \frac{\partial^2 M}{\partial x^2} \quad (\text{Equation 2})$$

From the above two equations 1 and 2, we get

$$\frac{\partial^2 M}{\partial x^2} = \rho A \frac{\partial^2 y}{\partial t^2} \quad (\text{Equation 3})$$

We know from strength of materials that

$$M = -EI \frac{\partial^2 y}{\partial x^2}$$

$$\text{So } \frac{\partial^2 M}{\partial x^2} = -EI \frac{\partial^4 y}{\partial x^4} \quad (\text{Equation 4})$$

Comparing equation 3 and 4 we get,

$$\frac{\partial^4 y}{\partial x^4} + \left( \frac{\rho A}{EI} \right) \frac{\partial^2 y}{\partial t^2} = 0 \quad (\text{Equation 5})$$

This is the general equation for transverse vibration. Thus the natural frequency can be found out by this theory as,

$$\omega_n = C * \sqrt{\frac{EI}{\rho A l^4}} \quad (\text{Equation 6})$$

Where,

- |  |                                       |
|--|---------------------------------------|
| E= Young's modulus of the material,      | I= Moment of inertia,                 |
| A= Area of cross section,                | l=length of the beam,                 |
| C= Constant depending mode of vibration, | C <sub>1</sub> =0.56 for first mode,  |
| C <sub>2</sub> = 3.52 for second mode,   | C <sub>3</sub> = 9.82 for third mode. |

The moment of inertia can be found out by relation,

$$I = \frac{bd^3}{12} \quad (\text{Equation 7})$$

Where,

b= width of the beam, d= depth of the beam.

Due to presence of crack, moment of inertia of the beam changes and correspondingly the natural frequency also changes. For a constant beam material and cross section the reduced moment of inertia will be found by relation below.

$$I_1 = I - I_c \quad (\text{Equation 8})$$

Where,

$I_1$ =Moment of inertia of a cracked beam,  $I$ =Moment of inertia of Uncracked beam,

$I_c$ =Moment of inertia of cracked beam element.

Thus by the use of equation 6, 7 and 8 we can find out the different modes of natural frequencies for the cantilever beam.

### MATERIAL GEOMETRY

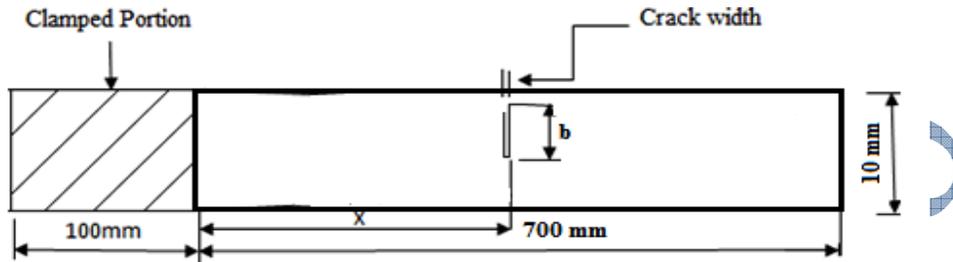


Fig.4 Cracked Square Beam Specimen

Table 1 Different Beam models and their dimensions

Beam Model No.	Material	Cross section dimension (mm)	Cracked/Uncracked	Position and location of crack	
				Crack depth (mm)	Crack location (mm)
1	Structural Steel $E= 210 \times 10^9$ $N/m^2$ , $\rho = 7850 \text{ Kg/m}^3$ , length = 0.7m.	10×10	Uncracked	0	0
2		10×10	Cracked	1	175
3		10×10	Cracked	2	175
4		10×10	Cracked	1	350
5		10×10	Cracked	2	350

### SAMPLE CALCULATION FOR BEAM MODEL 1

Length= 700 mm, Cross-section= 10\*10 mm, Uncracked,  $\rho= 7850 \text{ Kg/mm}^3$

$$A= 100 \times 10^{-6} \text{ m}^2$$

$$I= (0.01) (0.01)^3 / 12 \text{ mm}^4$$

$$= 8.33 \times 10^{-10} \text{ m}^4$$

Now from equation 6,

$$\omega = C \times \sqrt{\frac{210 \times 10^9 \times 8.33 \times 10^{-10}}{7850 \times 100 \times 10^{-6} \times 0.7^4}}$$

$$\omega = C \times 30.47 \text{ rad/s.}$$

For first three modes we have the value of constant C,

Thus,

$$\omega_1 = 0.56 \times 30.47$$

$$\omega_1 = 17.06 \text{ rad/s.}$$

Similarly  $\omega_2$  and  $\omega_3$  will be

$$\omega_2 = 3.52 \times 30.47 = 107.22 \text{ rad/s}$$

$$\omega_3 = 9.82 \times 30.47 = 299.11 \text{ rad/s.}$$

Similarly Calculated for Beam model 2 and 3

**Table 2 Theoretical natural frequencies for the beam models by using Euler's Beam Theory**

Beam model no.	RCD	RCL	First Natural Frequency	Second Natural Frequency	Third Natural Frequency
1	0	0	17.06	107.22	299.11
2	0.2	0.25	17.04	107.14	298.82
3	0.4	0.25	16.98	106.76	297.84
4	0.2	0.5	17.04	107.14	298.82
5	0.4	0.5	16.98	106.76	297.84

## CONCLUSION

- It has been observed that the natural frequency changes substantially due to the presence of cracks depending upon location and size of cracks.
- When the crack positions are constant i.e. at particular crack location, the natural frequencies of a cracked beam are inversely proportional to the crack depth.
- It has been observed that the change in frequencies is not only a function of crack depth, and crack location, but also of the mode number.

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