

# BIANCHI TYPE I STRING COSMOLOGICAL MODEL IN CYLINDRICALLY SYMMETRIC INHOMOGENEOUS UNIVERSE

BARKHA R. TRIPATHI

Department Of Mathematics And Statistics, University College of Science,  
Mohan Lal Sukhadia University.Udaipur-313001 (Rajasthan), India.

ATUL TYAGI

Department Of Mathematics And Statistics, University College of Science,  
Mohan Lal Sukhadia University.Udaipur-313001 (Rajasthan), India.

SWATI PARIKH

Department Of Mathematics And Statistics, University College of Science,  
Mohan Lal Sukhadia University.Udaipur-313001 (Rajasthan), India.

## ABSTRACT

Cylindrically Symmetric Inhomogeneous Bianchi Type I String Cosmological Model is investigated. To get the deterministic solution, we assume the condition  $A = B$  where  $B = f(x)g(t)$  and  $C = h(x)k(t)$  on metric potentials. In general the model represents expanding, shearing and non rotating universe. The physical and geometric aspects of the model are also discussed.

**KEYWORDS:** Cosmic string , Inhomogeneous universe, Cylindrically Symmetric .

## INTRODUCTION

In the standard model the universe is assumed to be homogeneous and isotropic and the space-time is described by the FRW metric. But from observations we know that there are inhomogeneities on scales less than 150 Mpc. In order to understand the influence of these inhomogeneities on the expansion of the universe, it is important to study other solutions of Einstein equations that don't assume the homogeneity such as the Lemaitre-Tolman-Bondi metric or a more general Szekeres metric. The Lemaitre-Tolman-Bondi metric is well understood and it has been used to model in homogeneity ever since it was discovered by Lemaitre [19]. The Szekeres metric was studied quite recently by Krasinski [15,16] and it was used by Bolejko [7-10] to model structure formation. Since it has no symmetries it is possible to describe more structures within one model. If the universe is not homogeneous the cosmological data may depend on the position of the observer and it is important to perform proper averaging of physical quantities.

Cylindrically symmetric space time play an important role in the study of the universe on a scale in which anisotropy and inhomogeneity are not ignored. Inhomogeneous cylindrically symmetric cosmological models have significant contribution in understanding some essential features of the universe such as the formation of galaxies during the early stages of their evolution. Bali et al. [1,2 ] studied a Bianchi type VI magnetized barotropic bulk viscous fluid massive string universe and some exact solutions for a homogeneous Bianchi type VI<sub>0</sub> space time filled with a magnetized bulk viscous fluid in the presence of a massive cosmic string.

Bali and Tyagi [3] and Pradhan et al. [20] have investigated cylindrically symmetric inhomogeneous cosmological models in presence of electromagnetic field. Barrow and Kunze [4,5] found a wide class of exact cylindrically symmetric flat and open inhomogeneous string universe. In their solutions all physical quantities depend on at most one space coordinate and the time. The case of cylindrical symmetry is natural because of the mathematical simplicity of the field equations whenever there exists a direction in which the pressure equal to energy density.

Cosmic strings play an important role in the study of the early universe. These strings arise during the phase transition after the big bang explosion as the temperature goes down below some critical temperature as predicted by grand unified theories [11,12,13, 25]. It is believed that cosmic strings give rise to density perturbation which leads to formation of galaxies [26]. These cosmic strings have stress energy and couple to the gravitational fluid. Therefore, it is interesting to study the gravitational effect which arises from strings. The general treatment of strings was initiated by Letelier [17,18] and Stachel [24].

Baysal et al. [6] , Kilinc and Yavuz [14] ,Pradhan et.al [21] have investigated some string cosmological models in cylindrically symmetric inhomogeneous universe. Ribeiso and Sanyal [23] studied Bianchi type VI<sub>0</sub> models containing a viscous fluid in the presence of an axial magnetic field . Pradhan, Rai and Singh [22] have studied cylindrically symmetric inhomogeneous universe with electromagnetic field in string cosmology.

In this chapter , we have obtained Cylindrically Symmetric Inhomogeneous Bianchi Type I String Cosmological Model . To get the deterministic solution, we assume the condition  $A = B$  where  $B = f(x) g(t)$  and  $C = h(x) k(t)$  on metric potentials. In general the model represents expanding, shearing and non rotating universe. The physical and geometric aspects of the model are also discussed.

### THE METRIC AND FIELD EQUATIONS

We consider the metric in the form

$$ds^2 = A^2(dx^2 - dt^2) + B^2dy^2 + C^2dz^2 \quad \dots(1)$$

where A, B, and C are functions of x and t. The energy momentum tensor for the string is of the form

$$T_i^j = \rho u_i u^j - \lambda x_i x^j \quad \dots(2)$$

where  $u_i$  and  $x_i$  satisfy conditions,

$$u^i u_j = -x^i x_j = -1 \quad \dots(3)$$

and  $u^i x_j = 0$

Here  $\rho$  is the rest energy density of the cloud of strings,  $\lambda$  is the tension density of the strings. The unit space like vector  $x^i$  represents the string direction in the cloud, i.e the direction of anisotropy and the unit time like vector  $u^i$  describes the four velocity vector of the matter satisfying the following conditions

$$g_{ij} v^i v^j = -1 \quad \dots(4)$$

we assume that the coordinates system is co-moving and so that

$$v^j = (0,0,0, A^{-1}) \text{ and } x^j = (A^{-1}, 0,0,0) \quad \dots(5)$$

The Einstein's field equations  $\left( \text{with } \frac{8\pi G}{C^4} = 1 \right)$

$$R_i^j - \frac{1}{2} R g_i^j = -T_i^j \quad \dots(6)$$

The Einstein's field Equations (6) for the line element (1), lead to the following system of equations

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} - \frac{A_4}{A} \left( \frac{B_4}{B} + \frac{C_4}{C} \right) - \frac{A_1}{A} \left( \frac{B_1}{B} + \frac{C_1}{C} \right) - \frac{B_1 C_1}{BC} + \frac{B_4 C_4}{BC} = \lambda A^2 \quad \dots(7)$$

$$\left( \frac{A_4}{A} \right)_4 - \left( \frac{A_1}{A} \right)_1 + \frac{C_{44}}{C} - \frac{C_{11}}{C} = 0 \quad \dots(8)$$

$$\left( \frac{A_4}{A} \right)_4 - \left( \frac{A_1}{A} \right)_1 + \frac{B_{44}}{B} - \frac{B_{11}}{B} = 0 \quad \dots(9)$$

$$-\frac{B_{11}}{B} - \frac{C_{11}}{C} + \frac{A_1}{A} \left( \frac{B_1}{B} + \frac{C_1}{C} \right) + \frac{A_4}{A} \left( \frac{B_4}{B} + \frac{C_4}{C} \right) - \frac{B_1 C_1}{BC} + \frac{B_4 C_4}{BC} = \rho A^2 \quad \dots(10)$$

$$\frac{B_{14}}{B} + \frac{C_{14}}{C} - \frac{A_4}{A} \left( \frac{B_1}{B} + \frac{C_1}{C} \right) - \frac{A_1}{A} \left( \frac{B_4}{B} + \frac{C_4}{C} \right) = 0 \quad \dots(11)$$

Where the sub-indices 1 and 4 in A, B, C and elsewhere denote ordinary differentiation with respect to x and t respectively.

### SOLUTION OF FIELD EQUATIONS:

The field equations 7-11 are five equations with five unknowns A, B, C, λ, ρ. We solve them in following ways.

By addition of equation (8) and (9)

$$2 \left[ \frac{A_4}{A} \right]_4 - 2 \left[ \frac{A_1}{A} \right]_1 + \frac{B_{44}}{B} - \frac{B_{11}}{B} + \frac{C_{44}}{C} - \frac{C_{11}}{C} = 0 \quad \dots (12)$$

Here to get a determinate solution, Let us consider

$$A = B \text{ where } B = f(x) g(t) \text{ and } C = h(x) k(t) \quad \dots (13)$$

Using equation (13) in equation (12), we have

$$3 \frac{B_{44}}{B} - 2 \frac{B_4^2}{B^2} + \frac{C_{44}}{C} = 3 \frac{B_{11}}{B} - 2 \frac{B_1^2}{B^2} + \frac{C_{11}}{C} = n \quad \dots(14)$$

From equations (11) and (13), we have

$$\frac{B_{14}}{B} + \frac{C_{14}}{C} - \frac{A_4 B_1}{AB} - \frac{A_1 C_4}{AB} - \frac{A_4 B_1}{AB} - \frac{A_1 C_4}{AB} = 0 \quad \dots(15)$$

$$\frac{B_{14}}{B} + \frac{C_{14}}{C} - \frac{2B_1 B_4}{B^2} - \frac{B_1 C_4}{BC} - \frac{B_4 C_1}{BC} = 0 \quad \dots(16)$$

Using equation (13) in (16), we have

$$\frac{h_1}{h} \left( \frac{k_4}{k} - \frac{g_4}{g} \right) - \frac{f_1}{f} \left( \frac{k_4}{k} + \frac{g_4}{g} \right) = 0 \quad \dots(17)$$

$$\frac{h_1}{h} = \frac{k_4}{k} + \frac{g_4}{g} \quad \dots (18)$$

$$\frac{h}{f_1} = \frac{k}{k_4} - \frac{g}{g_4} = m \text{ (constant)}$$

$$h = \alpha f^m \quad \dots (19)$$

$$\frac{k_4}{k} + \frac{g_4}{g} = m \left( \frac{k_4}{k} - \frac{g_4}{g} \right) \quad \dots(20)$$

$$k = \beta g^\gamma \quad \dots (21)$$

Here α and β are integrating constant and  $\gamma = \frac{m+1}{m-1}$ .

From equation (14), we have

$$3 \frac{B_{44}}{B} - 2 \frac{B_4^2}{B^2} + \frac{C_{44}}{C} = n \quad \dots(22)$$

$$g g_{44} + r g_4^2 = d g^2 \quad \dots(23)$$

$$g = C_2 \frac{1}{r+1} \sinh^{\frac{1}{r+1}}(bt + t_0) \quad \dots(24)$$

Where  $b = \sqrt{d(r+1)}$

Again From equation (14), we have

$$3 \frac{B_{11}}{B} - 2 \frac{B_1^2}{B^2} + \frac{C_{11}}{C} = n \quad \dots(25)$$

$$f f_{11} + \delta f_1^2 = l f^2 \quad \dots(26)$$

$$f = C_5 \frac{1}{\delta+1} \sinh^{\frac{1}{\delta+1}}(kx + x_0) \quad \dots(27)$$

where  $k = \sqrt{l(\delta+1)}$

and  $x_0, C_5,$  and  $k$  are constants of integration.

Hence, we obtain

$$B = P \sinh^{\frac{1}{1+\delta}} kX \sinh^{\frac{1}{1+r}} bT = A \quad \dots(28)$$

Where  $X = Cx + x_0$        $T = bt + t_0$        $P = C_2 \frac{1}{1+r} C_5 \frac{1}{1+\delta}$

$$\text{And } C = Q \sinh^{\frac{m}{1+\delta}} kX \sinh^{\frac{\gamma}{1+r}} bT \quad \dots(29)$$

Where  $Q = \alpha\beta C_2 \frac{m}{1+\delta} C_3 \frac{\gamma}{1+r}$ ,  $X = kx + x_0$ ,  $T = bt + t_0$

After using suitable transformation of co-ordinates metric (1) reduces to,

$$ds^2 = B^2(dx^2 - dt^2 + dy^2) + C^2 dz^2$$

$$ds^2 = P^2 \sinh^{\frac{2}{1+\delta}} kX \sinh^{\frac{2}{1+r}} bT (dx^2 - dt^2 + dy^2) + Q^2 \sinh^{\frac{m}{1+\delta}} CX \sinh^{\frac{\gamma}{1+r}} bT dz^2 \quad \dots(30)$$

Which may be considered as Bianchi Type I String Cosmological Model In Cylindrically Symmetric Inhomogeneous Universe.

### SOME PHYSICAL AND GEOMETRICAL FEATURES

The physical and geometrical properties of the model are given as follows:

String tension density  $\lambda$  of the model is given by

$$\lambda = \frac{1}{B^2} \left[ \frac{(\gamma^2 - \gamma r - \gamma - r - 1)}{(1+r)^2} b^2 \coth^2 bT - \frac{(1-2m)k^2 \coth^2 kX}{(1+\delta)^2} \right] + \frac{b^2(1+\gamma)}{B^2(1+r)} \quad \dots(31)$$

The energy density  $\rho$  of the model is given by

$$\rho = \frac{1}{B^2} \left[ \frac{(1+2\gamma)}{(1+r)^2} b^2 \coth^2 bT - \frac{[(\delta+1)(m+1) - m^2]k^2 \coth^2 kX}{(1+\delta)^2} - \frac{k^2(1+m)}{(1+\delta)} \right] \quad \dots(32)$$

The Expansion scalar  $\theta$  of the model is given by

$$\theta = \frac{(2 + \gamma) b \coth bT}{B(r + 1)} \quad (33)$$

The shear scalar  $\sigma$  of the model is given by

$$\sigma^2 = \frac{1}{3B^2(r + 1)^2} (\gamma - 1)^2 b^2 \coth^2 bT \quad \dots \quad \dots(34)$$

The proper volume  $V$  of the model is given by

$$V^3 = A^2 B C$$

$$V^3 = P^3 Q \sinh^{\frac{3}{1+\delta}} kX \sinh^{\frac{3}{1+r}} bT \sinh^{\frac{m}{1+\delta}} kX \sinh^{\frac{\gamma}{1+r}} bT \quad \dots(35)$$

The deceleration parameter  $q$  of the model is given by

$$q = -\frac{3(m^2 + 1)(\tanh^2 bT - 1)}{2(2m - 1)^2} \quad \dots(36)$$

From equation (34) and (33), we obtain

$$\frac{\sigma}{\theta} = \frac{1}{\sqrt{3}} \frac{(\gamma - 1)}{(\gamma + 2)} = \text{constant}$$

The magnitude of rotation  $\omega$  is zero i.e.

$$\omega = 0$$

## CONCLUSION

The model (30) starts with big bang at  $T = 0$  and goes on expanding till  $T = \infty$  when  $\theta$  becomes zero. It is clear that as  $T$  increases, the ratio of the shear scalar  $\sigma$  and expansion  $\theta$  tends to finite value i.e.  $\frac{\sigma}{\theta} \rightarrow \text{constant}$ . Hence the model does not approach isotropy for large value of  $T$ . The fluid flow is irrotational and it is observed that as  $T \rightarrow 0$  and  $X \rightarrow 0$ , the energy density  $\rho \rightarrow \infty$  and as  $T \rightarrow \infty$  and  $X \rightarrow \infty$ ,  $\rho \rightarrow 0$ . As  $T \rightarrow 0$  and  $X \rightarrow 0$ , the string tension density  $\lambda \rightarrow \infty$  and as  $T \rightarrow \infty$  and  $X \rightarrow \infty$ ,  $\lambda \rightarrow 0$  therefore the string will be disappear from the universe at later time. Since the deceleration parameter  $q < 0$ , hence the model represents an accelerating universe. The model has point type singularity at  $T=0$  as when  $T \rightarrow 0$ ,  $g_{11} = g_{22} = g_{33} = 0$ . In general the model represents expanding, shearing and non rotating universe.

## REFERENCES

- 1) Bali,R., Pradhan ,A. and Hassan,A., (2008). *Bianchi Type VI<sub>0</sub> Magnetized Barotropic Bulk Viscous Fluid Massive String Universe in General Relativity. International Journal of Theoretical Physics*, 47, 2594-2604.
- 2) Bali, R. , Banerjee, R. and Banerjee, S.K. , (2008). *Bianchi Type VI<sub>0</sub> Magnetized Bulk Viscous Massive String Cosmological Model in General Relativity. Astrophysics and Space Science*, 317, 21-26.
- 3) Bali, R. and Tyagi, A. ,(1989). *A Cylindrically Symmetric Inhomogeneous Cosmological Model With Electromagnetic Field. General Relativity and Gravitation*.21, 797-806
- 4) Barrow, J. D. and Kunze, K. E. , (1997). *Spherical Curvature Inhomogeneities In String Cosmology . Physics Review D* 56, 741-752 .
- 5) Barrow, J. D. and Kunze, K. E. (1998). *A Wide Class of Exact Cylindrically Symmetri Flat And Open Inhomogeneous String Universe. Physics Review D*57, 2255 .

- 6) Baysal, H. , Yavuz, I. ,Tarhan, I. ,Camci, U. and I. Yilmaz, (2001). Some String Cosmological Models In Cylindrically Symmetric Inhomogeneous Universe. Turk Journal of Physics. 25, 283 -292.
- 7) Bolejko K., (2007). *Evolution Of Cosmic Structures In Different Environments In The Quasi Spherical Szekeres Model.* arXiv: astro-ph/0610292v2
- 8) Bolejko K.,(2008). *Volume Averaging In The Quasi Spherical Szekeres Model.* arXiv:0808.0376v2 [astro-ph].
- 9) Bolejko, K., Anderson,C.,(2008) *Apparent And Average Acceleration Of The Universe.* arXiv:0807.3577v2 [astroph].
- 10) Bolejko K.,(2006). *Structure Formation In The Quasi Spherical Szekeres Model.* arXiv:astro-ph/0604490v2.
- 11) Everett, A.E., (1981). *Cosmic Strings In Unified Gauge Theories Physics Review* 24, 858.
- 12) Kibble, T.W.B. (1976). *Topology of Cosmic Domains and Strings. Journal of Physics A. Mathematical and General.* 9, 1387.
- 13) Kibble, T.W.B.(1980) *Some Implications Of A Cosmological Phase Transition. Physics Reports* 67, 183-199.
- 14) Kilinc ,C. B. and Yavuz, I. (1996) . *Some String Cosmological Models In Cylindrically Symmetric Inhomogeneous Universe. Astrophysics and Space Science.* 238, 239-245.
- 15) Krasinski A., (2007). *Physical and Geometrical Interpretation of the  $q \leq 0$  Szekeres Models .* arXiv:0710.2171v1 [gr-qc].
- 16) Krasinski A., (2008). *Geometry And Topology Of The Quasi-Plane Szekeres Model.* arXiv:0805.0529v3 [gr-qc]
- 17) Letelier, P.S., (1979). *Physics Review D*20, 1249.
- 18) Letelier, P.S. , (1983). *String Cosmologies. Physics Review D*28, 2414-2419.
- 19) Lemaitre G., *L'Universe en Expansion, Ann. Soc. Sci. Bruxelles, A*53, 641-680, (1933)
- 20) Pradhan, A., Chakrabarty, I , and Saste, N. N. , (2001). *Magnetized Cylindrically Symmetric Universe With Bulk Viscosity. International Journal of Modern Physics D* 10, 741-750.
- 21) Pradhan, A., Mishra, M. K. , and Yadav, A. K. (2007). *A New Class of String Cosmological Models in Cylindrically Symmetric Inhomogeneous Universe.* gr-qc/0705.1765.
- 22) Pradhan, A. , Rai, A.,and Singh, S. K. , (2007) .*Cylindrically symmetric inhomogeneous universe with electromagnetic field in string cosmology. Astrophysics and Space Science.* 312, 261 – 265.

- 23) Ribeiro, B.M and A.K. Sanyal, (1987). *Bianchi-VI<sub>0</sub> Viscous Fluid Cosmology with Magnetic Field. Journal of Mathematical Physics*, 28, 657-660.
- 24) Stachel, J. (1980). *Thickening the string. I. The string perfect dust. Physical Review D*. 21, 2171.
- 25) Vilenkin, A. (1981). *Cosmic strings. Physical Review D*24, .2082- 2089.
- 26) Zel'dovich, Ya, B. (1980). *Cosmological fluctuations produced near a singularity. Mon, Not. R. Astron. Soc.* 192, 663- 667.

IJIERT